EECS 281: **Test 3 (5 pages)** **Due: Thursday, April 7, 2005**

Name: ________________________ Email: ______________________ Grade: ______ (100 points max)

1. (10%) Please answer the following True or False in the context of Boolean Algebra:

T F $\prod_{ab}(3) = \overline{\overline{a}\,\overline{b}}$

T F $ab + abc = ab + abd$

T F $\prod_{abc}(1,7,3,5,6) = \sum_{abc}(2,4,1,7,0)$

T F $\sum_{ab}(3) = \overline{\overline{a} + \overline{b}}$

T F $ab\overline{c} + abc = ab\overline{d} + abd$

2. (10%) Use Boolean Algebra to establish the identity. Show the Theorem numbers (i.e. T1-T13) for each step of your proof:

Theorem	Expression
	$1 = a(a+b) + (\overline{a}+b)(a+\overline{b}) + \overline{a\overline{b}}$

3a. (20%) Show the optimal minimal circling in the k-map in minterm function $\prod_{abcd}(2,3,5,7,8,9,13,15)$ in the left-hand figure below.

3a	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

Minimal k-map

3d	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

Static-1 hazard free k-map

3b. Give the MSOP in cube notation= ________________________________

3c. Give the MSOP in symbolic boolean algebra= ________________________________

3d. Show the optimal k-map designed to cover static-1 hazards in the right-hand figure above.

4a. (20%) Show the optimal multi-output minimal circling the terms and in the k-map in minterm function $F = \sum_{abcd} = (2,3,5,7,8,9,13,15)$ and $G = \sum_{abcd} = (2,3,4,6,8,9,12,14)$. Indicate which circle belongs to what function.

F	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

G	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

4b. Give the boolean algebra common term of multi-output MSOP = ________________________________

4c. Give the boolean algebra multi-output MSOP of F= ________________________________

4d. Give the boolean algebra multi-output MSOP of G= ________________________________

4e. Draw and fill in the PLA:

5a. (20%). Do the Quine-McCluskey Algorithm of $\sum_{a,b,c,d}(2,3,5,7,8,9,13,15)$.

Group	Minterms	0-cubes	Minterms	1-cubes	Minterms	2-cubes
G_0						
G_1						
G_2						
G_3						
G_4						

5b. Fill in the covering table

EPI?	PI-cubes										
	Covered?										

5c. Give the boolean algebra MSOP= ________________________________

6a. (10%) Given $\sum_{a,b,c,d}(2,3,5,7,8,9,13,15)$ and the don't cares (1,11,14), show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

6b. Give the boolean algebra MSOP of the k-map: ______________________________

7. (10%) A programmer has written the following C code fragment (assume variables are 1-bit):

```
f=0;
if ( (a | b) & c){
    if (b) { f=1; }
}
else if (a & b) { f=0; }
```

7a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

7b. Give the optimal k-map of 7a.

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$				
a				

7c. Give the boolean algebra MSOP of the k-map: ______________________________

7d. Re-write as optimal C code:

x1. (5% Extra credit) Write the C language for-loop for the recurrence equation, $t_n = 2t_n + n - 1$, where $t_0 = 2$.

x2. (10% Extra credit) Write the 8051 assembler for the recurrence equation of problem x1, use R0 for variable i, R1 for variable n, R2 for variable t.

Theorem	Relationship	Dual	XOR	Property
T1	$a1 = a$	$a + 0 = a$	$a \oplus 0 = a$	Identity
T2	$a0 = 0$	$a + 1 = 1$	$a \oplus 1 = \overline{a}$	Domination
T3	$aa = a$	$a + a = a$	$a \oplus a = 0$ $a \oplus a \oplus a = a$	Idempotency
T4	$\overline{\overline{a}}$			Involution
T5	$a\overline{a} = 0$	$a + \overline{a} = 1$	$a \oplus \overline{a} = 1$	Complement
T6	$ab = ba$	$a + b = b + a$	$a \oplus b = b \oplus a$	Commutative
T7	$(ab)c = a(bc)$	$(a + b) + c = a + (b + c)$	$(a \oplus b) \oplus c = a \oplus (b \oplus c)$	Associative
T8	$(a + b)(a + c) = a + bc$	$a(b + c) = ab + ac$	$a(b \oplus c) = ab \oplus ac$	Distributive
T9	$a(a + b) = a$	$a + ab = a$	$a \oplus ab = a\overline{b}$	Absorption Covering
T10	$(a + b)(a + \overline{b}) = a$	$ab + a\overline{b} = a$	$ab \oplus a\overline{b} = a$	Combining
T11	$(a + b)(\overline{a} + c)(b + c) = (a + b)(\overline{a} + c)$	$ab + \overline{a}c + bc = ab + \overline{a}c$		Consensus Proof by k-map
T12	$a + a + \cdots + a = a$	$aa \cdots a = a$	$a \oplus a \oplus \cdots \oplus a_{odd} = a$ $a \oplus a \oplus \cdots \oplus a_{even} = 0$	Generalized Idempotency
T13	$\overline{a + b} = \overline{a}\overline{b}$	$\overline{ab} = \overline{a} + \overline{b}$	$\overline{ab} = \overline{a} \oplus \overline{b} \oplus \overline{a}\overline{b}$	DeMorgan
XOR	$ab = a \oplus \overline{b} \oplus \overline{a}\overline{b}$	$a + b = a \oplus b \oplus ab$	$a \oplus b = \overline{a} \oplus \overline{b} = a\overline{b} + \overline{a}b$	Definition