EECS 281: **Sample Test 3 (5 pages)** **Due: Tuesday, November 2, 2004**

Name: ____________________ Email: ____________________ Grade: ______ (100 points max)

1. (10 points) Please answer the following True or False in the context of Boolean Algebra:

T F $\overline{a} + b = \overline{a}\overline{b} + b$ (hint: k-map or truth table)

T F $a\overline{a} = a \oplus a$ (Wakerly section 5.8.1 page 410-413)

T F $\sum_{ab}(1,2) = a \oplus b$ (hint: use truth table)

T F $a = ab + \overline{b}a$

T F $\sum_{abc}(0,2,4,5,6) = a\overline{b} + \overline{c}$ (hint: use k-map)

T F $b + \overline{b} = \overline{a\overline{a}}$

T F $\sum_{abc}(1,3,5,7) = \prod_{abc}(2,4,6)$

T F $a + \overline{a}b = a + b$

T F $\overline{a + \overline{a}} = \overline{c}c$ (see MIT problem set 1 #22)

T F $\prod_{abc}(1,2,5) = (a+b+\overline{c})(a+\overline{b}+c)(\overline{a}+b+\overline{c})$ (Wakerly, pg 208)

2. (10 points) Use Boolean Algebra to establish the identity. Show the Theorem numbers (i.e. T1-T13) for each step of your proof:

Theorem	Expression
	$c = \overline{(b+\overline{c})(a+\overline{c})} + cb + c\overline{b}$

3a. (15 points) Show the optimal mimimal circling in the k-map in minterm function $f(a,b,c,d) = (\overline{a}\oplus b)+\overline{a}cd+bcd$ (hint: replace xor as $a \oplus b$ with $a\overline{b}+\overline{a}b$):

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

3b. Give the $\sum_{abcd} =$ ________________________________

3c. Give MSOP= ________________________________

4a. (20 points) Show the optimal multi-output mimimal circling the terms and in the k-map in minterm function $F = \sum_{abcd} = (4, 12, 13, 15)$ and $G = \sum_{abcd} = (6, 13, 14, 15)$. Indicate which circle belongs to what function.

F	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

G	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

4b. Give the common term of multi-output MSOP = ________________________________

4c. Give the multi-output MSOP of F= ________________________________

4d. Give the multi-output MSOP of G= ________________________________

4e. Fill in the PLA

5a. (20 points). Do the Quine-McCluskey Algorithm of $\sum_{a,b,c,d}(0, 1, 6, 7, 14, 15)$.

Group	Minterms	0-cubes	Minterms	1-cubes	Minterms	2-cubes
G_0						
G_1						
G_2						
G_3						
G_4						

5b. Fill in the covering table

EPI?	Needed?	PI-cubes							
		Covered?							

5c. Give the MSOP= ________________________________

5d. Show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

5e. Give the MSOP of the k-map: ________________________________

6a. (10 points) Given $\sum_{a,b,c,d}(0,1,6,7,14,15)$ and the don't cares (2, 8, 10), show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$				
ab				
$a\bar{b}$				

6b. Give the MSOP of the k-map: ________________________________

7. (15 points) A programmer as written the following C code fragment:

```
f=0;
if (a ˆ b) {
     if (c) { f=1; }
}
else if (b | c) { f=1; }
else if (∼ b) { f=1; }
```

7a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

7b. Give the optimal k-map of 7a.

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$				
a				

7c. Give the MSOP of the k-map: ________________________________

7d. Re-write as optimal C code:

x1. (Extra credit 10 points) Using C++ data types for a **machine that uses a char of 4-bits**, convert the following into **one's** complement big-endian binary and if not, then show why not?: where **signed char s, a=6, b=−3;** For addition and indicate if end-around-carry, overflow and/or carry has occurred. Show work.

Give unsigned char range:	
Give signed char range:	Wakerly Table 2-6, page 40
unsigned char x = 2;	Wakerly section 2.5.6 page 38
signed char x = −2;	Wakerly section 2.5.6 page 38
s = (∼ a)+1;	
s = ∼ a;	
s = -b;	Hint: one's complement, not two's complement
s = a & b;	bitwise operation
s = a + b;	end-around-carry occurs here! Given in Wakerly section 2.7, page 44
s = a - b;	Hint: complement and then add

Theorem	Relationship	Dual	XOR	Property
T1	$a1 = a$	$a + 0 = a$	$a \oplus 0 = a$	Identity
T2	$a0 = 0$	$a + 1 = 1$	$a \oplus 1 = \overline{a}$	Domination
T3	$aa = a$	$a + a = a$	$a \oplus a = 0$ $a \oplus a \oplus a = a$	Idempotency
T4	$\overline{\overline{a}}$			Involution
T5	$a\overline{a} = 0$	$a + \overline{a} = 1$	$a \oplus \overline{a} = 1$	Complement
T6	$ab = ba$	$a + b = b + a$	$a \oplus b = b \oplus a$	Commutative
T7	$(ab)c = a(bc)$	$(a + b) + c = a + (b + c)$	$(a \oplus b) \oplus c = a \oplus (b \oplus c)$	Associative
T8	$(a + b)(a + c) = a + bc$	$a(b + c) = ab + ac$	$a(b \oplus c) = ab \oplus ac$	Distributive
T9	$a(a + b) = a$	$a + ab = a$	$a \oplus ab = a\overline{b}$	Absorption Covering
T10	$(a + b)(a + \overline{b}) = a$	$ab + a\overline{b} = a$	$ab \oplus a\overline{b} = a$	Combining
T11	$(a + b)(\overline{a} + c)(b + c) = (a + b)(\overline{a} + c)$	$ab + \overline{a}c + bc = ab + \overline{a}c$		Consensus Proof by k-map
T12	$a + a + \cdots + a = a$	$aa \cdots a = a$	$a \oplus a \oplus \cdots \oplus a_{odd} = a$ $a \oplus a \oplus \cdots \oplus a_{even} = 0$	Generalized Idempotency
T13	$\overline{a + b} = \overline{a}\overline{b}$	$\overline{ab} = \overline{a} + \overline{b}$	$\overline{ab} = \overline{a} \oplus \overline{b} \oplus \overline{a}\overline{b}$	DeMorgan
XOR	$ab = a \oplus \overline{b} \oplus \overline{a}\overline{b}$	$a + b = a \oplus b \oplus ab$	$a \oplus b = \overline{a} \oplus \overline{b} = a\overline{b} + \overline{a}b$	Definition