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EECS 281:	Test 3 (5 pages)	Due: Tuesday, November 2, 2004
Name: ______________	Email: ______________	Grade: ____ (100 points max)

1. (10 points) Please answer the following True or False in the context of Boolean Algebra:

T F $\bar{b} + a = \bar{a}\bar{b} + a$ $\quad \bar{b} + a = \bar{b}(a+\bar{a}) + \bar{a} = \bar{b}a + \bar{a}\bar{b} + a = a(1+\bar{b}) + \bar{a}\bar{b}$
$= a + \bar{a}\bar{b} = RHS$

T F $\sum_{ab}(1,2) = \bar{a} \oplus \bar{b}$ $\quad : \bar{a} \oplus \bar{b} = \bar{a}\bar{\bar{b}} + \bar{b}\bar{\bar{a}} = \bar{a}b + a\bar{b} = \sum_{ab}(1,2)$

T F $\sum_{abcd}(1,8,4,2,11,13,14,7) = \prod_{abcd}(3,5,6,9,10,12,0)$ $\quad$ RHS $\sum_{abcd} = \prod_{abcd}(3,5,6,9,10,12,0,15)$

T F $\prod_{abc}(0,4,7) = (\bar{a}+\bar{b}+\bar{c})(a+b+c)(\bar{a}+b+c)$

T F $\bar{a}(b+c) = \bar{a} + \bar{a}b + \bar{a}c$ $\quad$ LHS $= \bar{a}b + \bar{a}c$, RHS $= \bar{a} + \bar{a}b + \bar{a}c$ $\}$ cannot be simplified

2. (10 points) Use Boolean Algebra to establish the identity. Show the Theorem numbers (i.e. T1-T13) for each step of your proof:

Theorem	Expression
	$a = \overline{(\bar{a}+\bar{c})(\bar{a}+c)} + (a+b)(a+c)a$
	RHS $= \overline{(\bar{a}+\bar{c})(\bar{a}+c)} + (a+b)(a+c)a$
	$= \overline{(\bar{a}+\bar{c})} + \overline{(\bar{a}+c)} + (a+b)(a+c)a$
	$= (\bar{\bar{a}} \cdot \bar{\bar{c}}) + (\bar{\bar{a}} \cdot \bar{c}) + (a+b)(a+c)a$
	$= a \cdot c + a \cdot \bar{c} + (a+b)(a+c)a$
	$= a(c+\bar{c}) + a(a+b)(a+c)$
	$= a + a(a+b)(a+c)$
	$= a[1 + (a+b)(a+c)]$
	$= a[1]$
	$= a =$ LHS

3 a) $a\bar{c}\bar{d} = a\bar{c}\bar{d}(b+\bar{b}) = ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d}$

$\bar{c} \oplus d = \bar{c}\bar{d} + cd = \bar{c}\bar{d}(b+\bar{b}) + cd(b+\bar{b})$

$= b\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d} + bcd + \bar{b}cd$

$= b\bar{c}\bar{d}(a+\bar{a}) + \bar{b}\bar{c}\bar{d}(a+\bar{a}) + bcd(a+\bar{a})$
$+ \bar{b}cd(a+\bar{a})$

$= ab\bar{c}\bar{d} + \bar{a}b\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + abcd$
$+ \bar{a}bcd + a\bar{b}cd + \bar{a}\bar{b}cd$

$\bar{b} = \bar{b}(a+\bar{a})$

$= a\bar{b} + \bar{a}\bar{b}$

$= a\bar{b}(c+\bar{c}) + \bar{a}\bar{b}(c+\bar{c})$

$= a\bar{b}c + a\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}\bar{b}\bar{c}$

$= a\bar{b}c[d+\bar{d}] + a\bar{b}\bar{c}[d+\bar{d}] + \bar{a}\bar{b}c[d+\bar{d}]$
$+ \bar{a}\bar{b}\bar{c}[d+\bar{d}]$

$= a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}cd + \bar{a}\bar{b}c\bar{d}$
$+ \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d}$

$f(a,b,c,d) = \sum(0,1,2,3,4,7,8,9,10,11,12,15)$

3a. (15 points) Show the optimal mimimal circling in the k-map in minterm function $f(a,b,c,d) = a\bar{c}\bar{d} + (\bar{c} \oplus d) + \bar{b}$.

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1	1	1	1
$\bar{a}b$	1		1	
ab	1		1	
$a\bar{b}$	1	1	1	1

3b. Give the $\sum_{abcd} =$ $= \Sigma(0,1,2,3,4,7,8,9,10,11,12,15)$

3c. Give MSOP= $\bar{b} + \bar{c}\bar{d} + cd = \bar{b} + (\bar{c} \oplus d)$

4a. (20 points) Show the optimal multi-output mimimal circling the terms and in the k-map in minterm function $F = \sum_{abcd} = (0,2,4,8,10)$ and $G = \sum_{abcd} = (0,2,6,8,10)$. Indicate which circle belongs to what function.

common circles

F	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1			1
$\bar{a}b$	1			
ab				
$a\bar{b}$	1			1

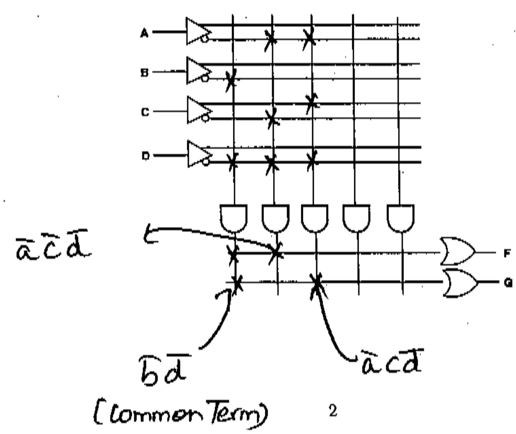
G	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1			1
$\bar{a}b$				1
ab				
$a\bar{b}$	1			1

4b. Give the common term of multi-output MSOP = $\bar{b}\bar{d}$

4c. Give the multi-output MSOP of F= $\bar{b}\bar{d} + \bar{a}\bar{c}\bar{d}$

4d. Give the multi-output MSOP of G= $\bar{b}\bar{d} + \bar{a}c\bar{d}$

4e. Fill in the PLA



$\Sigma(a,b,c,d)\ (0,5,15,10,2,8)$

G0	✓0	0000
G1	✓2	0010
	✓8	1000
G2	5	0101
	✓10	1010
G4	15	1111

✓(0,2)	00–0
✓(0,8)	–000
✓(2,10)	–010
✓(8,10)	10–0

✓(0,2,8,10)	–0–0
✓(0,8,2,10)	–0–0
(0,2,8,10)	–0–0

$f = \bar{b}\bar{d} + abcd + \bar{a}b\bar{c}d$

5a. (20 points) Do the Quine-McCluskey Algorithm of $\sum_{a,b,c,d}(0, 5, 15, 10, 2, 8)$.

Group	Minterms	0-cubes	Minterms	1-cubes	Minterms	2-cubes
G_0	✓0	0000	✓0,2 ✓0,8	00-0 -000	✓0,2,8,10 ✓0,2,8,10	-0-0 -0-0
G_1	✓2 ✓8	0010 1000	✓2,10 ✓8,10	-010 10-0		
G_2	5 ✓10	0101 1010				
G_3						
G_4	15					

5b. Fill in the covering table

EPI?	Needed?	PI-cubes	0	5	15	10	2	8	
✓	Yes	0101		✓					
✓	Yes	1111			✓				
✓	Yes	-0-0	✓			✓	✓	✓	
		Covered?	✓	✓	✓	✓	✓	✓	

5c. Give the MSOP= $\bar{b}\bar{d} + abcd + \bar{a}b\bar{c}d$

5d. Show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1			1
$\bar{a}b$		1		
ab			1	
$a\bar{b}$	1			1

5e. Give the MSOP of the k-map: $\bar{b}\bar{d} + \bar{a}b\bar{c}d + abcd$

$$a \wedge b \wedge c$$

$$= ((a \oplus b) \oplus c))$$

$$= (a\bar{b} + \bar{a}b) \oplus c$$

$$= \overline{(a\bar{b} + \bar{a}b)}\, c + \bar{c}\,(a\bar{b} + \bar{a}b)$$

$$= \left[(\bar{a} + \bar{\bar{b}}) \cdot (\bar{\bar{a}} + \bar{b})\right] c + a\bar{b}\bar{c} + \bar{a}b\bar{c}$$

$$= \left[(\bar{a} + b)\,c \cdot (a + \bar{b})\,c\right] + \left[a\bar{b}\bar{c} + \bar{a}b\bar{c}\right]$$

$$= \left[(\bar{a}c + bc) \cdot (ac + \bar{b}c)\right] + \left[a\bar{b}\bar{c} + \bar{a}b\bar{c}\right]$$

$$= abc + \bar{a}\bar{b}c + a\bar{b}\bar{c} + \bar{a}b\bar{c}$$

6a. (10 points) Given $\sum_{a,b,c,d}(0,5,15,10,2,8)$ and the don't cares (7, 13), show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1			1
$\bar{a}b$		1	d	
ab		d	1	
$a\bar{b}$	1			1

6b. Give the MSOP of the k-map: $\bar{b}\bar{d}+bd=(\overline{b\oplus d})$

7. (15 points) A programmer as written the following C code fragment:

```
f=1;
if (~a) {
    if (~(b ^ c)) { f=0; }
}
else if (b ^ c) { f=0; }
```

7a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\sum_{abc} f = (1,2,4,7)$

b	c	b ^ c	~(b ^ c)
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

7b. Give the optimal k-map of 7a

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$		1		1
a	1		1	

7c. Give the MSOP of the k-map: $\bar{a}\bar{b}c+\bar{a}b\bar{c}+a\bar{b}\bar{c}+abc$ [Cannot be further minimized from k-Map]

7d. Re-write as optimal C code:

f=((~a&~b&c)|(~a&b&~c)|(a&~b&~c)|(a&b&c))

or the best Solution would be f=(a^b^c)

7e. (5 points extra credit) Re-write as optimal C code using minimal parenthesis and as many exclusive-or's as possible:

f= a^b^c

x1. (10 points extra credit) Using C++ data types for a **machine that uses a char of 5-bits**, convert the following into **one's** complement big-endian binary and if not, then show why not?: where **signed char** s, a=−1, b=6; For addition and indicate if end-around-carry, overflow and/or carry has occurred. Show work.

Give unsigned char range:	0 to $2^5 \Rightarrow 0$ to 32
Give signed char range:	$-(2^{n-1}-1)$ to $(2^{n-1}-1) = -15$ to $+15$
unsigned char x = 30;	11110
signed char x = −1;	$-1 \Leftrightarrow \underline{1}1110$
s = (~ a)+1;	$\Rightarrow$ 00010
s = ~ a;	$\Rightarrow$ 00001
s = −b;	$-6 \Rightarrow \underline{1}1001$
s = a & b;	$\Rightarrow$ 00110
s = a + b;	$\Rightarrow \underline{0}\,0101 \Rightarrow +5$
s = a − b;	$-7 \Rightarrow -1-6 = \underline{1}1000$

Theorem	Relationship	Dual	XOR	Property
T1	$a1 = a$	$a+0=a$	$a \oplus 0 = a$	Identity
T2	$a0 = 0$	$a+1=1$	$a \oplus 1 = \overline{a}$	Domination
T3	$aa = a$	$a+a=a$	$a \oplus a = 0$ $a \oplus a \oplus a = a$	Idempotency
T4	$\overline{\overline{a}}$			Involution
T5	$a\overline{a} = 0$	$a+\overline{a}=1$	$a \oplus \overline{a} = 1$	Complement
T6	$ab = ba$	$a+b=b+a$	$a \oplus b = b \oplus a$	Commutative
T7	$(ab)c = a(bc)$	$(a+b)+c = a+(b+c)$	$(a \oplus b) \oplus c = a \oplus (b \oplus c)$	Associative
T8	$(a+b)(a+c) = a+bc$	$a(b+c) = ab+ac$	$a(b \oplus c) = ab \oplus ac$	Distributive
T9	$a(a+b) = a$	$a+ab=a$	$a \oplus ab = a\overline{b}$	Absorption Covering
T10	$(a+b)(a+\overline{b}) = a$	$ab + a\overline{b} = a$	$ab \oplus a\overline{b} = a$	Combining
T11	$(a+b)(\overline{a}+c)(b+c) = (a+b)(\overline{a}+c)$	$ab+\overline{a}c+bc = ab+\overline{a}c$		Consensus Proof by k-map
T12	$a+a+\cdots+a=a$	$aa\cdots a = a$	$a \oplus a \oplus \cdots \oplus a_{odd} = a$ $a \oplus a \oplus \cdots \oplus a_{even} = 0$	Generalized Idempotency
T13	$\overline{a+b} = \overline{a}\overline{b}$	$\overline{ab} = \overline{a}+\overline{b}$	$\overline{ab} = \overline{a} \oplus \overline{b} \oplus \overline{a}\overline{b}$	DeMorgan
XOR	$ab = a \oplus \overline{b} \oplus \overline{a}\overline{b}$	$a+b = a \oplus b \oplus ab$	$a \oplus b = \overline{a} \oplus \overline{b} = a\overline{b} + \overline{a}b$	Definition